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# Green-Schwarz superstring on $AdS_3 \times S^3$ *

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Abstract: Green-Schwarz action of Type IIB string on $AdS_3 \times S^3$ is constructed via coset superspace approach. The construction relies exclusively on symplectically Majorana-Weyl spinor formalism, thus permitting it easier to prove $\kappa$-symmetry for on-shell Type IIB supergravity backgrounds.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

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## Contents





## 1. Introduction

Recently, prompted by the observation that anti-de Sitter supergravity is dual to conformal field theory at large $N$ and large 't Hooft coupling [1], there has arisen renewed interest to study string dynamics in curved spacetime. According to the correspondence, string worldsheet and loop corrections correspond to inverse 't Hooft coupling and $1/N$ corrections in the dual conformal field theory. As such, it should be important to develop a systematic method that would enable construction of superstring action on a curved background and quantization thereof. For the anti-de Sitter spaces under consideration, the background curvature is induced by Ramond-Ramond tensor field flux threaded through compact space. This has led one to investigate string and M-theory on anti-de Sitter space, including fundamental string [2, 3] or $p$-brane propagation [4, 5] and Type IIB $SL(2, \mathbf{Z})$ duality [6], a subject which has been left largely unexplored until recently.

For the background $AdS_5 \times S^5$, a Green-Schwarz superstring action has been constructed recently [2]. The construction is based on supercoset space approach and exhibits manifest $\kappa$-symmetry for on-shell Type IIB supergravity backgrounds. Although this marks a significant progress, $\kappa$-symmetry gauge-fixing and quantization thereof still seem to pose considerable difficulties. Therefore, it would be most desirable to study the superstring propagation on a simpler Ramond-Ramond background.

One of the simpler backgrounds is Type IIB string on $AdS_3 \times S^3 \times M_4$, where $M_4$ denote a Ricci-flat four-dimensional compact manifold such as $T^4$ or $K3$. In fact, this background arises as the near-horizon geometry of Type IIB D1-D5 brane configuration [1] (wrapped on $T^4$ or $K_3$), which threads nonzero Ramond-Ramond (RR) tensor flux through $M_4$. By performing $SL(2, \mathbf{Z})$ S-duality rotation, which is now part of U-duality group in six-dimensions, the configuration is mapped to Type IIB F1-NS5 brane configuration [7]. The six-dimensional background is now $AdS_3 \times S^3$ threaded by (Neveu-Schwarz)-(Neveu-Schwarz) (NS-NS) tensor field flux. For $M^4 = T^4$ or $K^3$, on which the D1-D5 or F1-NS5 branes are wrapped on, the six-dimensional superstring carries sixteen supersymmetries, half as much as the ten-dimensional Type IIB superstring. Clearly, from six-dimensional $(2,0)$ supergravity point of view, both cases correspond to a consistent on-shell background, on which a non-critical superstring can propagates. One novelty of these backgrounds is that the symmetry group $SO(2,2)$ of the $AdS_3$ space is realized on the dual conformal field theory as an infinite-dimensional Virasoro algebra.

Recently, the Green-Schwarz superstring action on $AdS_3 \times S^3$ has been constructed by Pesando and by Rahmfeld and Rajaraman [8]. It turns out that their formulations are not the most convenient ones for investigation of certain interesting issues such as consistency check with on-shell background of six-dimensional $(2,0)$ supergravity [9], which has been formulated exclusively using the symplectically Majorana-Weyl $d = 2$spinors, and U-duality transformation.

In the present paper, utilizing symplectically Majorana-Weyl $d = 6$ spinor formalism, we provide a new formulation of the Green-Schwarz superstring action on $AdS_3 \times S^3$. As shown in section 2, the symplectically Majorana-Weyl $d = 6$ spinors descends in fact from the Majorana-Weyl $d = 10$ spinors when decomposed on the above background. With the new formalism, one can prove in a straightforward way the $\kappa$-symmetry constraints to on-shell $(2,0)$ supergravity background. The formalism also enables to investigate straightforwardly the Type IIB $SL(2, \mathbf{Z})$ duality transformation, which is now a subgroup of six-dimensional U-duality transformation. In section 3, employing the coset superspace approach, we then construct the Green-Schwarz action for a fundamental (non-critical) string propagating on $AdS_3 \times S^3$, on which nonzero R-R (but not NS-NS ) tensor field flux is threaded.



## 2. Symplectically Majorana-Weyl spinor decomposition

Before we dwell on construction of the Green-Schwarz superstring action, it is useful to recapitulate decomposition of the $d = 10$ Type IIB spinors into spinors on the product space under consideration.[1] In the Type IIB string, there are thirty-two supercharges, consisting of two sixteen component, Majorana-Weyl spinors $\eta_L, \eta_R$. Here, $L, R$ subscript denotes their origin from the left- and the right-movers on the world-sheet, viz, the spacetime spinors are counted after chiral doubling of the string worldsheet is taken into account. We adopt convention of the spacetime signature to be $(- + \cdots +)$. The $d = 10$ $(32 \times 32)$ Dirac matrices are denoted by $\Gamma_M$. The charge

[1] Related analysis have been considered in [10, 11].

conjugation $\psi_c$ of a spinor $\psi$ is given by

$$\psi^{\mathrm{T}}\mathcal{C} \equiv \overline{\psi} = \psi^\dagger\Gamma^0 \,. \tag{2.1}$$

The charge conjugation matrix $\mathcal{C}$ then satisfies the following relations:

$$\mathcal{C}\Gamma_M\mathcal{C}^{-1} = -{\Gamma_M}^{\mathrm{T}} \,, \qquad \mathcal{C}^{\mathrm{T}} = -\mathcal{C} \,. \tag{2.2}$$

Because the $\eta_{L,R}$ are sixteen-component Majorana-Weyl spinors, they satisfy both the Majorana conditions

$${\eta_L}^{\mathrm{T}}\mathcal{C} = \overline{\eta}_L \,, \qquad {\eta_R}^{\mathrm{T}}\mathcal{C} = \overline{\eta}_R \,, \tag{2.3}$$

and the Weyl conditions

$$\Gamma^{11}\eta_L = +\eta_L \,, \qquad \Gamma^{11}\eta_R = +\eta_R \,. \tag{2.4}$$

For the present case, the $AdS_3 \times S^3 \times M^4$ background threaded by nontrivial RR-flux is generated by a supersymmetric combination of ground-state $N_1$ D-strings and $N_5$ D5-branes. Suppose that the D1-strings are oriented along 1-direction and the D5-branes along 16789-directions, where 6789-directions are along $M^4$. Due to open string sector attached to these branes, unbroken supersymmetry is determined by spinor parameters satisfying

$$\Gamma^{01}\eta_L = +\eta_R \,, \qquad \Gamma^{01}\eta_R = +\eta_L \tag{2.5}$$

from the D1-strings, and

$$\Gamma^{01}\eta_L = +\eta_R \,, \qquad \Gamma^{016789}\eta_R = +\eta_L \tag{2.6}$$

from the D5-branes. Combining the two conditions, we find that $\eta_L, \eta_R$ should satisfy

$$\Gamma_\perp\eta_L = +\eta_L \,, \qquad \Gamma_\perp\eta_R = +\eta_R \,, \qquad \left(\Gamma_\perp \equiv \Gamma^{6789}\right) . \tag{2.7}$$

Since the spinor projection is independent of transverse six-dimensional spacetime, the chirality condition eq. (2.7) can be rephrased as follows. If we decompose the sixteen-component Majorana-Weyl spinors $\eta_{L,R}$ into $(AdS_3 \times S_3)$ and $M_4$ parts, they are given by

$$\eta_{L,R} = \theta^{\alpha,\alpha'} \otimes \epsilon^{ia}_{L,R} \,, \tag{2.8}$$

where $\alpha, \alpha'$ are $Spin(3)$, $Spin(3)'$ indices and $i, a$ are $Spin(4)$, $Spin''(3)$ indices. They are associated to $(AdS_3 \times S^3)$ and to $M_4$ times extra 'isospin' spaces, respectively. Now, for both $M^4 = T^4$ and $K_3$, residual supersymmetry of the background is possible only when the the spinors $\epsilon^{ia}_{L,R}$ are chirally projected precisely as in eq. (2.7).

In the case of S-dual background configuration, viz $N_1$ F-strings along 1-direction and $N_5$ NS5-branes along 16789-directions, the unbroken supersymmetry parameters are characterized by

$$\Gamma^{01}\eta_L = +\eta_L\,, \qquad \Gamma^{01}\eta_R = -\eta_R \tag{2.9}$$

due to F-string and

$$\Gamma^{016789}\eta_L = +\eta_L\,, \qquad \Gamma^{016789}\eta_R = -\eta_R \tag{2.10}$$

due to NS5-brane. Combining the two conditions, one find again exactly the same chiral projection condition as in eq. (2.7).

In fact, the Type IIB S-duality mapping between the above two configurations is realized on the two spinors as a $SO(2)$ rotation. To see this, it would be convenient to combine the two Majorana-Weyl spinors into two Weyl spinors:

$$\eta = \eta_L + i\eta_R, \qquad \eta^* = \eta_L - i\eta_R\,. \tag{2.11}$$

In terms of the two complexified spinors, the supersymmetry projection condition eq. (2.7) is rewritten as

$$\Gamma_\perp \eta = +\eta, \quad \Gamma_\perp \eta^* = +\eta^*\,, \tag{2.12}$$

for both D1-D5 and F1-NS5 systems. Under the S-duality, the complexified spinors are rotated by a $U(1)$-phase:

$$S\,:\,\eta \to e^{+i\pi/4}\eta\,, \qquad \eta^* \to e^{-i\pi/4}\eta^*\,. \tag{2.13}$$

Actually, this is the special case of general spinor transformation rule under S-duality mapping [12] :

$$\eta \to \exp\left(\frac{i}{2}\arg(c\tau + d)\right)\eta \qquad \text{for} \qquad \tau \to \frac{a\tau + b}{c\tau + d} \quad (ad - bc = 1) \tag{2.14}$$

and oppositely for $\eta^*$, where $\tau = \frac{a_{\rm RR}}{2\pi} + ie^{-\phi}$ is the Type IIB dilaton.

Closely related observation has been made in the study of the Type IIB D-brane actions and their transformation under the $SL(2, \mathbf{Z})$ mapping on both flat [13] and $AdS^5 \times S^5$ [6] spaces. Essential to the analysis of D-brane action was the fact that the S-duality mapping acts on the two $d = 10$ Majorana-Weyl spinors as a rotation on $SO(2)$ subgroup of the $SL(2, \mathbf{R})$, the classical duality symmetry of Type IIB superstring. Likewise, for similar of dual D-brane action on $AdS_3 \times S^3$, it should become essential to take into consideration of the spinor transformation rule eqs. (2.13, 2.14).

## 3. Green-Schwarz string action on $AdS_3 \times S_3$

We now construct the Green-Schwarz superstring action on $AdS_3 \times S^3$ via super-group manifold approach and utilizing $d = 6$ symplectically Majorana-Weyl spinors.[2]

The $AdS_3$ space possesses $SU(1,1|2)$ invariance as a graded spacetime symmetry, inherited from the isometry of Type IIB D1-D5 or F1-NS5 brane configuration. We would like to construct the Green-Schwarz superstring action which exhibits this graded symmetry manifestly. This is most conveniently achieved by viewing the the Green-Schwarz superstring action as a nonlinear sigma model on a super-group manifold. For the present case, the coset superspace is $\mathcal{G} = \frac{SU(1,1|2)_L \times SU(1,1|2)_R}{SO(1,2) \times SO(3)}$ whose even part is $\frac{(SU(1,1) \times SU(2))^2}{SO(1,2) \times SO(3)} \simeq \frac{SO(2,2)}{SO(1,2)} \times \frac{SO(4)}{SO(3)} \simeq AdS^3 \times S^3$.

The essential structure of $d = 6$ supercharges are the same as in the previous section. Starting from thirty-two component Majorana-Weyl spinors $\eta_L, \eta_R$ in ten dimensions, one first compactify on $M_4$. For D1-D5 brane configuration wrapped on $M_4 = T_4$ or $K_3$ (whose volume is taken very small), we have seen that the compactification projects each spinors chirally. Low-energt dynamics on the noncompact $d = 6$ spacetime is described precisely by (2,0) supergravity. The supergravity contains variety of tensor multiplets, as required by the gravitational anomaly cancellation (twenty-one if $M^4 = K3$). These tensor multiplets descend from NS-NS and R-R tensor fields in ten dimensions, but we will be restricting foregoing discussions only to the case for which only R-R tensor fields are turned on.

In six-dimensions with (2,0) supersymmetry, we use the symplectically Majorana-Weyl spinors , where the symplectic condition acts on $USp(4) = SO(5)$ R-symmetry indices $m, n$'s (and suppressing $d = 6$ spinor indices):

$$\theta_m^{\mathrm{T}} \Omega_{mn} = \overline{\theta}_n \equiv i \theta_n^\dagger \Gamma^0 \,. \tag{3.1}$$

Here, we take $d = 6$ $(8 \times 8)$ Dirac matrices to be antisymmetric and the charge conjugation matrix $\mathcal{C}$ symmetric. As shown in the previous section, this is achieved by decomposing the symmetric ten-dimensional Dirac matrices into a tensor product of antisymmetric $M_4$ Dirac matrices and antisymmetric $d = 6$ ones. Likewise, antisymmetric charge conjugation matrix can be decomposed into a tensor product of antisymmetric $M_4$ matrix and symmetric $d = 6$ one. This way, the noncompact six-dimensional part whose background is described by (2,0) supergravity becomes completely decoupled from the internal space $M_4$. It now remains to decompose the noncompact six-dimensional spinors further into direct product of spinors on $AdS_3$ and $S^3$. First, the $SO(1,5)$ spinor indices are decomposed into $SO(1,2) \times SO(3)$. In doing so, the $d = 6$ R-symmetry $USp(4)$ is broken into $USp(2) \times USp(2)$, as we will see below. Acting on each of the two $USp(2)$ symplectically Majorana-Weyl spinors,

[2]The Green-Schwarz superstring propagates on the on-shell background of the truncated $d = 6$ $(2,0)$ supergravity theory and hence the string may be viewed as an "effective" non-critical string.

we decompose the $d = 6$ Dirac matrices as

$$\begin{aligned} \Gamma^a &= \gamma^a \otimes \mathbf{I} \otimes \sigma^1\,, \qquad & \mathcal{C} &= C_{2,1} \otimes C_3 \otimes \sigma^1 \\ \Gamma^{a'} &= \mathbf{I} \otimes \gamma^{a'} \otimes \sigma^2\,, \qquad & \Gamma^7 &= \mathbf{I} \otimes \mathbf{I} \otimes (-\sigma^3)\,. \end{aligned} \tag{3.2}$$

Here, $C_{2,1}, C_3$ denote charge conjugation matrices for $AdS_3$ and $S^3$ spaces respectively. The resulting two-component $SO(1,2)$ spinors are Majorana and the two-component $SO(3)$ spinors are pseudo-symplectic Majorana spinors. In this decomposition, the symplectic-Majorana condition can be written as

$$\bar{\theta}_{\alpha\alpha'\alpha''} \equiv \theta^{\beta\beta'\beta''} C_{\alpha\beta} C'_{\alpha'\beta'} \varepsilon_{\alpha''\beta''}\,. \tag{3.3}$$

The charge conjugation matrix are used in lowering and rasing the indices. We denote $SO(1,2)$ vector indices by $a, b, c$ and $SO(3)$ vector indices by $a', b', c'$. We use hatted indices $\hat{a}$ to denote the combination $(a, a')$. The generators of $so(1,2)$ and $so(3)$ Clifford algebras are $2 \times 2$ matrices $\gamma_a$ and $\gamma_{a'}$

$$\gamma^{(a}\gamma^{b)} = \eta^{ab} = (-++)\,, \qquad \gamma^{(a'}\gamma^{b')} = \eta^{a'b'} = (+++)\,. \tag{3.4}$$

We now introduce two symplectically Majorana-Weyl supercharges $Q^I, I = 1, 2$. We also denote the Pauli matrices by $\tau_i, i = 1, 2, 3$. Then, the $SU(1,1|2)_L \times SU(1,1|2)_R$ supersymmetry algebra is given by

$$\begin{aligned} [P_a, P_b] &= J_{ab}\,, \\ [P_{a'}, P_{b'}] &= -J_{a'b'}\,, \\ [P_a, J_{bc}] &= \eta_{ab} P_c - \eta_{ac} P_b\,, \\ [P_{a'}, J_{b'c'}] &= \eta_{a'b'} P_{c'} - \eta_{a'c'} P_{b'}\,, \\ [J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} + 3\,\text{terms}\,, \\ [J_{a'b'}, J_{c'd'}] &= \eta_{b'c'} J_{a'd'} + 3\,\text{terms}\,, \\ [Q_I, P_a] &= \frac{1}{2}\tau_{3IJ} Q_J \gamma_a\,, \qquad\qquad (3.5) \\ [Q_I, P_{a'}] &= \frac{i}{2}\tau_{3IJ} Q_J \gamma_{a'}\,, \\ [Q_I, J_{ab}] &= -\frac{1}{2} Q_I \gamma_{ab}\,, \qquad\qquad (3.6) \\ [Q_I, J_{a'b'}] &= -\frac{1}{2} Q_I \gamma_{a'b'}\,, \\ \{Q^I_{\alpha\alpha'\alpha''}, Q^J_{\beta\beta'\beta''}\} &= \delta_{IJ}(-2iC_{\alpha'\beta'}(C\gamma^a)_{\alpha\beta} P_a + 2C_{\alpha\beta}(C'\gamma^{a'})_{\alpha'\beta'} P_{a'})\varepsilon_{\alpha''\beta''} - \\ &\quad -i\tau_{3IJ}(C_{\alpha'\beta'}(C\gamma^{ab})_{\alpha\beta} J_{ab} - C_{\alpha\beta}(C'\gamma^{a'b'})_{\alpha'\beta'} J_{a'b'})\varepsilon_{\alpha''\beta''}\,. \qquad (3.7) \end{aligned}$$

Note that both sides of eq. (3.7) are symmetric under the exchange of $(\alpha\alpha'\alpha''I) \leftrightarrow (\beta\beta'\beta''J)$ since $C$ and $C'$ are antisymmetric while $C\gamma^a$, $C'\gamma^{a'}$, $C\gamma^{ab}$ and $C'\gamma^{a'b'}$ are all symmetric. In eq. (3.5) $\tau_{3IJ}$ factor is related to that in eq. (3.7) by the Jacobi identity. If we define $J_a \equiv \frac{1}{2}\varepsilon_{abc}J_{bc}$ and $J_{a'} \equiv \frac{1}{2}\varepsilon_{a'b'c'}J_{b'c'}$, then eq. (3.5) and eq. (3.6) can be written as

$$\left[Q_I, J^J_a\right] = \frac{1}{2}\delta_{IJ}Q_J\gamma_a\,, \tag{3.8}$$

$$\left[Q_I, J^J_{a'}\right] = -\frac{i}{2}\delta_{IJ}Q_J\gamma_{a'}\,, \tag{3.9}$$

where $J^{1,2}_a \equiv (J_a \pm P^a)/2$ and $J^{2,1}_{a'} \equiv (J_{a'} \pm P_{a'})/2$ with $P^a = \eta_{ab}P_b$.[3] The resulting new variables satisfy $SU(1,1)$ and $SU(2)$ algebras respectively:

$$\left[J^I_a, J^J_b\right] = -\delta^{IJ}\varepsilon^c{}_{ab}J^J_c\,,$$

$$\left[J^I_{a'}, J^J_{b'}\right] = -\delta^{IJ}\varepsilon_{a'b'c'}J^J_{c'}\,.$$

Now, the eq. (3.7) can be rewritten into a transparent form:

$$\{Q^I_{\alpha\alpha'\alpha''}, Q^J_{\beta\beta'\beta''}\} = -4\tau_{3IJ}(iC_{\alpha'\beta'}(C\gamma^a)_{\alpha\beta}J^J_a + C_{\alpha\beta}(C'\gamma^{a'})_{\alpha'\beta'}J^J_{a'})\varepsilon_{\alpha''\beta''}\,. \tag{3.10}$$

viz, we have shown explicitly that the superalgebra under consideration is indeed $SU(1,1|2)_L \times SU(1,1|2)_R$. From the consideration of the Jacobi identity of the form $[Q,\{Q,Q\}]+$ cyclic permutations, we obtain the following useful identity:

$$\gamma^a\psi_{(1}\bar{\psi}_2\gamma_a\psi_{3)} - \gamma^{a'}\psi_{(1}\bar{\psi}_2\gamma^{'}_a\psi_{3)} = 0 \tag{3.11}$$

for arbitrary symplectically Majorana-Weyl spinors $\psi_i, i = 1,2,3$. This identity can be proven using the Fierz identities.[4] Moreover, in the scaling limit $P_{\hat{a}} \to RP_{\hat{a}}$, $J_{\hat{a}} \to J_{\hat{a}}$ and $Q^I \to \sqrt{R}Q^I$ with $R \to \infty$, we obatin the supersymmetric algebra in flat six-dimensional space. In this limit, the superstring action on $AdS^3 \times S^3$ should be reduced to the well-known form in the flat space. Note that the supersymmetry algebra in flat space has $USp(4)$ R-symmetry whereas on $AdS^3 \times S^3$ this is broken to $USp(2) \times USp(2)$. This is because of the term containing $\tau_{3IJ}$.

The Mauer-Cartan equations are given by

$$dL^a = -L^b \wedge L^{ba} - i\bar{L}^I\gamma^a \wedge L^I\,, \tag{3.12}$$
$$dL^{ab} = -L^a \wedge L^b - L^{ac} \wedge L^{cb} - i\bar{L}^I\tau_{3IJ}\gamma^{ab}L^J\,,$$
$$dL^{a'} = -L^{b'} \wedge L^{b'a'} + \bar{L}^I\gamma^{a'} \wedge L^I\,, \tag{3.13}$$
$$dL^{a'b'} = +L^{a'} \wedge L^{b'} - L^{a'c'} \wedge L^{c'b'} + i\bar{L}^I\tau_{3IJ}\gamma^{a'b'}L^J\,,$$
$$dL^I = +\frac{1}{2}\tau_3^{IJ}\gamma^aL^J \wedge L^a + \frac{1}{4}\gamma^{ab}L^I \wedge L^{ab} + \frac{i}{2}\tau_3^{IJ}\gamma^{a'}L^J \wedge L^{a'} + \frac{1}{4}\gamma^{a'b'}L^I \wedge L^{a'b'}, \tag{3.14}$$
$$d\bar{L}^I = -\frac{1}{2}\tau_3^{IJ}\bar{L}^J\gamma^a \wedge L^a - \frac{1}{4}\gamma^{ab}\bar{L}^I \wedge L^{ab} - \frac{i}{2}\tau_3^{IJ}\bar{L}^J\gamma^{a'} \wedge L^{a'} - \frac{1}{4}\bar{L}^I\gamma^{a'b'} \wedge L^{a'b'}\,.$$

[3] In proving this we have used the identities $\gamma_{ab} = -\varepsilon^c{}_{ab}\gamma_c$ and $\gamma_{a'b'} = i\varepsilon_{a'b'c'}\gamma_{c'}$.

[4] Similar identity can be derived from the consideration of six-dimensional super Yang-Mills theory with the symplectically Majorana-Weyl spinors. The identity obtained this way has been useful in proving the $\kappa$-symmetry of the $d = 6$ superstring action on flat background. The same remarks should be applicable to the superstring action on $AdS^3 \times S^3$ background as well.

Consider the expression $\mathcal{H}_1 \equiv A^{IJ} L^a \bar{L}^I \gamma^a L^J$. We find that

$$
\begin{aligned}
d\mathcal{H}_1 = & -i\bar{L}^K \gamma^a L^K A^{IJ} \bar{L}^I \gamma_a L^J + \frac{1}{2}(\tau_3 A + A\tau_3)_{IK} L^a L^b \bar{L}^I \gamma^{ab} L^K + \\
& + \frac{i}{2}(A\tau_3 - \tau_3 A)^{IK} L^a L^{a'} \bar{L}^I \gamma^a \gamma^{a'} L^K \,.
\end{aligned} \tag{3.15}
$$

Similarly, for $\mathcal{H}_2 \equiv A'^{IJ} L^{a'} \bar{L}^I \gamma^{a'} L^J$, we have

$$
\begin{aligned}
d\mathcal{H}_2 = & \bar{L}^K \gamma^{a'} L^K A'^{IJ} \bar{L}^I \gamma_{a'} L^J + \frac{1}{2}\left(\tau_3 A' + A'\tau_3\right)_{IK} L^{a'} L^{b'} \bar{L}^I \gamma^{a'b'} L^K + \\
& + \frac{1}{2}\left(A'\tau_3 - \tau_3 A'\right)^{IK} L^{a'} L^a \bar{L}^I \gamma^a \gamma^{a'} L^K \,.
\end{aligned} \tag{3.16}
$$

If we choose $A, A'$ to be anticommuting with $\tau_3$ and $A' = iA$, we find that all terms cancel except for the four-fermion terms. Moreover, if $A$ is proportional to $\tau_2$, then $\mathcal{H}_1$ and $\mathcal{H}_2$ themselves vanish as $\bar{L}^I \gamma^{\hat{a}} L^J = \bar{L}^J \gamma^{\hat{a}} L^I$. Thus, the only choice is that $A$ is proportional to $\tau_1$. For this choice, the four-fermion terms also vanish due to the identity eq. (3.11).

Thus the Green-Schwarz action on $AdS_3 \times S^3$ is given by

$$
I = -\frac{1}{2}\int_{\partial M_3} d^2\sigma \sqrt{-g}\, g^{ij}\left(L_i^a L_j^a + L_i^{a'} L_j^{a'}\right) + i\int_{M_3} \mathcal{H} \,, \tag{3.17}
$$

where $\mathcal{H}$ is a closed 3-form invariant under $SO(1,2) \times SO(3)$:

$$
\mathcal{H} = \tau_1^{IJ}\left(L^a \wedge \bar{L}^I \gamma^a \wedge L^J + iL^{a'} \wedge \bar{L}^I \gamma^{a'} \wedge L^J\right). \tag{3.18}
$$

The relative coefficient between the usual kinetic term and the Wess-Zumino term is fixed by the $\kappa$-symmetry. Under the variation $\delta\theta^I$,

$$
\begin{aligned}
\delta L^a &= 2i\bar{L}^I \gamma^a \delta\theta^I \,, \\
\delta L^{a'} &= -2\bar{L}^I \gamma^{a'} \delta\theta^I \,, \\
\delta L^I &= d\delta\theta^I + \frac{1}{2}\tau_{3IJ}\left(L^a \gamma^a \delta\theta^J + iL^{a'} \gamma^{a'} \delta\theta^J\right) + \\
& \quad + \frac{1}{4}L_{ab}\gamma^{ab}\delta\theta^I + \frac{1}{4}L'_{ab}\gamma^{ab'}\delta\theta^I \,.
\end{aligned} \tag{3.19}
$$

The variation of $\mathcal{H}$ is given by $\delta\mathcal{H} = d\Lambda$, where

$$
\Lambda = \tau_{1IJ}\left(2L^a \bar{L}^I \gamma^a \delta\theta^I + 2iL^{a'} \bar{L}^I \gamma^{a'} \delta\theta^I\right). \tag{3.20}
$$

In order to identify the $\kappa$-symmetry, we have found it is useful to define "rotated" variables :

$$\theta^{1'} \equiv \frac{\theta^1 + \theta^2}{\sqrt{2}}\,, \qquad \theta^{2'} \equiv \frac{-\theta^1 + \theta^2}{\sqrt{2}} \tag{3.21}$$

and analogous expressions for other spinors.

If we define $\delta x^a \equiv \delta X^M L_M^a$, $\delta x^{a'} \equiv \delta X^M L_M^{a'}$, and $\delta\theta^I \equiv \delta X^M L_M^I$, the $\kappa$-symmetry transformation is given by

$$\begin{aligned}
\delta_\kappa x^a &= 0\,, && (3.22)\\
\delta_\kappa x^{a'} &= 0\,, && (3.23)\\
\delta_\kappa \theta^{I'} &= 2\left(L_i^a \gamma^a - i L_i^{a'} \gamma^{a'}\right) \kappa^{iI'} && (3.24)\\
\delta_\kappa(\sqrt{g} g^{ij}) &= -16 i \sqrt{g}\left( P_-^{jk} \bar{L}_k^{1'} \kappa^{i1'} + P_+^{jk} \bar{L}_k^{2'} \kappa^{i2'} \right),
\end{aligned}$$

where $P_\pm = \frac{1}{2}\left(g^{ij} \pm \frac{1}{\sqrt{g}}\varepsilon^{ij}\right)$. We see that the eigenbasis for the $\kappa$-symmetry does not coincide with the diagonal basis for the superconformal algebra.

The gauge-fixing of the $\kappa$-symmetry may be proceeded similarly as in ref. [3]. The algebraic steps and results are essentially the same, and will not be repeated here. However, it's not clear if this Killing spinor gauge fixing provides any useful insight to understand the quantization of the theory.

Using the above super-coset approach, we can find the closed form expression for the supervielbein by solving the Mauer-Cartan equations eqs. (27 - 29). For $AdS^5 \times S^5$, such an expression has been found in [14]. Similar analysis was made for M2-brane on $AdS^4 \times S^7$ or $AdS^7 \times S^4$ [5]. In [5], the result obtained from the super-coset approach was also compared to the construction of the super-space vielbein in terms of the component field of eleven-dimensional on-shell supergravity. The two results turn out to agree perfectly. For the $d = 2$ (2,0) supergravity, we expect essentially the same result. The only difference is that the $d = 6$ (2,0) supergravity is described in $SO(5, N_T)$ invariant way for the tensor multiplet coupling [9]. For example, if we consider the Type IIB compactification on $K3$, the resulting theory has $SO(5, 21)$ invariance which rotates the tensor multiplets coming from the compactification. If we compare the Killing spinor equation eq. (16) in [15] with our expression on the commutator [Q, P] in eq. (20), we see that in our expression the $SO(5, N_T)$ symmetry is broken by choosing a particular direction in the $SO(5, N_T)$ space.

So far, we have considered the $AdS^3 \times S^3$ with R-R 3-form field. In general, the background 3-form flux can be a mixture of NS-NS and R-R field. If the $AdS^3 \times S^3$ background includes the NS-NS 3-form flux, in the Wess-Zumino term, we should have an additional term proportional to $L^a \wedge L^b \wedge L^c H_{abc}$. If the 3-form flux comes entirely from NS-NS field, the corresponding action with this additional term is nothing but the Green-Schwarz formulation of the Type IIB string considered by [7] using the Neveu-Schwarz-Ramond formalism. It would be very intersting if one can establish equivalence between the Green-Schwarz formalism and the Neveu-Schwarz-Ramond formalism for $AdS^3 \times S^3$ background with pure NS-NS 3-form flux.



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## 4. Conclusion

In this paper, we have constructed the Green-Schwarz superstring action on $AdS_3 \times S^3$, where the $S^3$ is threaded by pure Ramond-Ramond tensor flux. Essential to the construction was to utilize the symplectically Majorana-Weyl representation of the $d = 6$ supercharges. The representation makes it most transparent and straightforward to compare the $\kappa$-symmetry constraint to on-shell background of the standard $(2, 0)$ supergravity.

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